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Two-Dilaton Theories in Two Dimensions from Dimensional Reduction

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Abstract

Dimensional reduction of generalized gravity theories or string theories generically yields dilaton fields in the lower-dimensional effective theory. Thus at the level of D=4 theories, and cosmology many models contain more than just one scalar field (e.g. inflaton, Higgs, quintessence). Our present work is restricted to two-dimensional gravity theories with only two dilatons which nevertheless allow a large class of physical applications.

The notions of factorizability, simplicity and conformal simplicity, Einstein form and Jordan form are the basis of an adequate classification. We show that practically all physically motivated models belong either to the class of factorizable simple theories (e.g. dimensionally reduced gravity, bosonic string) or to factorizable conformally simple theories (e.g. spherically reduced Scalar-Tensor theories). For these theories a first order formulation is constructed straightforwardly. As a consequence an absolute conservation law can be established.

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1 Introduction

Dilaton fields have experienced an impressive comeback in recent years in a broad range of gravitational theories. Motivated by their appearance in string theories scalar fields play an increasingly important role in modern physics. In the context of those theories, but also as a feature of any higher-dimensional theory of gravity, the concept of compactification has become a standard method in many models that leads inevitably to the occurrence of dilatons in the reduced action. It is the aim of this paper to discuss some common properties of two-dimensional two-dilaton theories (TDT) where at least one dilaton field is produced by dimensional reduction.

Historically the dilaton was introduced for the first time by Kaluza and Klein who proposed a five-dimensional gravity theory (KKT) to unify general relativity with electrodynamics [1]. The scalar field created by reduction to 4 dimensions inspired Fierz [2] and Jordan [3] to invent the first Scalar-Tensor theory (STT) in 4D where the dilaton field was interpreted as a local gravitational coupling "constant". Already Fierz [2] investigated the connection of this theory with usual general relativity by conformal transformations. Later work of Brans and Dicke [4] revived the theory which in the following will be called Jordan-Brans-Dicke theory (JBD). Recently the interest in STT has increased enormously due to the observation of accelerating galaxies with high redshift, indicating a positive cosmological constant [5]. Again the transformation of that constant into a scalar field ("quintessence") has been proposed [6].

A one-dilaton theory in two dimensions emerges naturally [7] in connection with spherically reduced general relativity (SRG). The reformulations of general one-dilaton theories in D=2 as first order theories with torsion [8] have led to various new insights, including e.g. the discovery of a conservation law [9]. These results have been extended to the case of SRG with a massless scalar field minimally coupled in the 4D theory, i.e. the Einstein massless Klein-Gordon model (EMKG) [10, 11].

To motivate the interest in TDT we briefly summarize already existing models that belong to this category:

- The most obvious example is spherically reduced EMKG. Whenever one deals with a scalar field in ordinary general relativity and demands spherical symmetry one arrives at a TDT in 2D, where the 4D scalar field may be interpreted as one of the two dilatons.

- The polarized Gowdy model [12] is based on the existence of two commuting space-like Killing fields in a closed Einstein universe. Then

toric reduction directly leads to a TDT. This example is of particular interest because here both dilatons in the 2D theory are part of four-dimensional geometry. The polarized Gowdy model (in contrast to SRG) allows to retain one degree of freedom of the gravity waves which is transferred into one of the dilatons.

- Another example is given by KKT. Having already one dilaton in 4 dimensions one ends up again with a TDT in 2D through spherical reduction. This is not equivalent to spherically reduced EMKG since the Kaluza dilaton in the four-dimensional theory couples non-minimally to gravity.

- STT and nonlinear gravity theories are mainly rooted in KKT, thus their connection to TDT is very similar. In addition it has been shown that nonlinear gravity theories are formally equivalent to STT (cf. e.g. [13]).

Finally, TDT in 2D may serve as useful toy models for TDT in 4D which may be necessary to describe the appearance of various scalar fields, encountered in cosmology (Higgs, inflaton, stringy dilaton(-s), quintessence). Up to now there exists no 4D STT with a single scalar field playing the role of *all* of these fields and certain dimensional reductions of e.g. a 11D supergravity [14] can yield such theories by analogy to the $5 \to 4 \to 2$ reduction of spherically reduced KKT.

Thus, a dilaton field either may be produced by dimensional reduction (like in the effective theories for the Gowdy model or KKT) or it is introduced in the action as a generic scalar field (like in STT or the EMKG).

In section 2 we present the general framework of the TDT. The notions of Einstein form and Jordan form are introduced. As examples three significant physical applications are shown to fit into this framework.

In section 3 a useful classification scheme is invented distinguishing models that are simple and/or factorizable. We further investigate how conformal transformations affect these properties.

Section 4 is devoted to (conformally) simple factorizable theories. They can be treated in a first order form where a conservation law easily can be derived. Finally we examine the scaling properties of the conserved quantity.

In the Conclusions a table summarizes the various models we consider, together with their properties. Finally possible further applications are discussed.

2 General framework

The ansatz for the TDT

$$
\begin{aligned}
S_a &= \int_{M_2} d^2x\sqrt{-g}\,[V_0(X,Y)R + V_1(X,Y)\nabla_\alpha X\nabla^\alpha X + V_2(X,Y)\nabla_\alpha Y\nabla^\alpha Y \\
&\quad + V_3(X,Y)\nabla_\alpha X\nabla^\alpha Y + V_4(X,Y) + V_5(X,Y)f_m(S_n,\nabla_\alpha S_n,\dots)] \qquad (1)
\end{aligned}
$$

satisfies diffeomorphism invariance in 2D and the following requirements:

- Two scalar dilaton fields X, Y should appear in the 2D action.
- The action should be linear in the scalar curvature R since terms with higher power in R could be accommodated by modifying the arbitrary functions in (1) just like in theories with only one dilaton field [13]. In order to have a nontrivial dilaton geometry the factor V_0 is assumed never to be a constant.
- The dilaton fields' first derivatives enter quadratically multiplied by an arbitrary function of the dilatons (V_1 and V_2); in general, there will be a mixing between them ($V_3 \neq 0$).
- In addition, there is an arbitrary function of the dilaton fields, V_4, henceforth called "potential".
- Finally, there are contributions from one or more "matter fields" S_n which couple non-minimally to the dilatons whenever $V_5 \neq$ const.

Our paper will mainly deal with the special case $V_5 = 0$, for simplicity, although when conformal transformations are discussed we will have to reconsider the matter part since the coupling function V_5 will change in general, having important implications for geodesics and hence for the global structure of the manifold.

2.1 Standard forms

In the following we restrict the function $V_0(X,Y)$ further, in accordance with our intention to examine dimensionally reduced gravity models. The application of conformal transformations in the higher-dimensional theory reveals two standard forms, which have the advantage that all models considered in the Introduction fit into one of them. Bearing in mind that at least one of the dilatons (here called X) possesses a conformal weight $\alpha \neq 0$ (being part of the higher-dimensional metric), we cannot reach a form where $V_0 = const.$ through this kind of conformal transformation.

2.1.1 Einstein form

We call the first standard form "Einstein form"(EF) because it contains as the most important representative SRG in the Einstein frame in D=4.

It is given by

$$S_E = \int_{M_2} d^2x\sqrt{-g}\,\big[XR + V_1^E(X,Y)\nabla_\alpha X\nabla^\alpha X + V_2^E(X,Y)\nabla_\alpha Y\nabla^\alpha Y$$
$$+V_3^E(X,Y)\nabla_\alpha X\nabla^\alpha Y + V_4^E(X,Y) + V_5^E(X,Y)f_m(S_n,\nabla_\alpha S_n,\dots)\big] \quad (2)$$

and contains important special cases listed in table 1 at the end of the paper. A trivial subclass are one-dilaton theories where $V_i^E = V_i^E(X)$ and $V_2^E = V_3^E = V_5^E = 0$.

2.1.2 The Jordan form

Motivated by spherically reduced JBD in the Jordan frame, we call the second standard form "Jordan form" (JF). It reads

$$S_J = \int_{M_2} d^2x\sqrt{-g}\,\big[XYR + V_1^J(X,Y)\nabla_\alpha X\nabla^\alpha X + V_2^J(X,Y)\nabla_\alpha Y\nabla^\alpha Y$$
$$+V_3^J(X,Y)\nabla_\alpha X\nabla^\alpha Y + V_4^J(X,Y) + V_5^J(X,Y)f_m(S_n,\nabla_\alpha S_n,\dots)\big]\,. \quad (3)$$

Indeed its most important representatives are general spherically reduced STT. There $V_1^J = Y/(2X)$, $V_2^J = -wX/Y$, $V_3^J = 2$, $V_4 = -2Y + X\hat{V}(Y)$ and $V_5 = X$. In JBD $w = const.$ is called "Dicke parameter". $\hat{V}(Y)$ is a scalar potential (which vanishes in JBD) and V_5 is chosen such that it amounts to minimal coupling of matter fields in the Jordan frame of four-dimensional STT.

2.2 Applications: Specific models

In this section we consider three significant models somewhat more in detail. They are all constructed through dimensional reduction of D-dimensional gravity theories by assuming the existence of (D-2) spacelike Killing fields. In the first case we start from the spherically symmetric D-dimensional Einstein-Hilbert action with one (in D dimensions minimally coupled) massless scalar field. In the second case we apply the spherical reduction scheme to STT in D = 4 without matter where the scalar field plays the role of one of the two dilatons in two dimensions. In a last example we reduce the pure 4D polarized Gowdy model that has cylindrical symmetry and therefore one gravitational degree of freedom.

2.2.1 Spherically reduced Einstein gravity with massless scalar field

The D-dimensional (D≥4) Einstein-Hilbert action including a massless scalar field Y reads

$$S = \int_{M_D} d^D x \sqrt{-g} \left[R^{(D)} - \kappa \nabla_\mu Y \nabla^\mu Y \right]. \tag{4}$$

In D=4 Einstein gravity the constant κ is taken to be $16\pi G$ with Newton's constant G. If the D-dimensional spacetime M_D is spherically symmetric, its metric can be written as[1]

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta - X^2 (r,t)\, g_{\kappa\lambda} dx^\kappa dx^\lambda, \tag{5}$$

where $g_{\alpha\beta}$ is the metric of a two-dimensional Lorentz manifold M_2, $g_{\kappa\lambda}$ the metric of a (D-2)-sphere and X the dilaton field. The scalar curvature $R^{(D)}$ of M_D can be decomposed as (cf. e.g. [7])

$$R^{(D)} = R - \frac{(D-2)(D-3)}{X^2} \left[1 + \nabla_\alpha X \nabla^\alpha X\right] - 2(D-2)\frac{\Box X}{X} \tag{6}$$

where R on the right side is the curvature of M_2. To integrate out the isometric angular coordinates on the unit sphere S^{D-2} we only have to substitute the scalar curvature by the above expression and the measure by

$$\sqrt{(-1)^{D+1} g_{M_D}} = \sqrt{-g}\, \sqrt{(-1)^D g_{S^{D-2}}} \cdot X^{D-2}. \tag{7}$$

For later convenience we perform a field redefinition

$$X \to (D-2) X^{1/(D-2)}. \tag{8}$$

Up to a constant factor, the effective 2D action thus reads

$$S = \int\limits_{M_2} d^2 x \sqrt{-g} \left[XR + \frac{(D-3)}{(D-2)} \frac{(\nabla X)^2}{X} - \frac{(D-3)}{(D-2)} X^{\frac{(D-4)}{(D-2)}} - \kappa X (\nabla Y)^2 \right] \tag{9}$$

and obviously is of the Einstein form (2).

[1] We use the metric signature $(+,-,\dots)$. The indices α, β, γ take the values $0, 1$ whereas the indices κ, λ run from 2 to $D-1$. Indices μ, ν run from 0 to $D-1$.

2.2.2 Spherically reduced Scalar-Tensor theories

The four-dimensional STT action without matter is given by ($\phi \in \mathbb{R}^+$)

$$S_{STT} = \int_{M_4} d^4x\sqrt{-g}\left[\phi R - w(\phi)\frac{\nabla_\mu\phi\nabla^\mu\phi}{\phi} + V(\phi)\right]. \tag{10}$$

Here ϕ is the (positive) scalar field (STT field) that couples non-minimally to the metric. In the original KKT this non-trivial coupling was a result from a former reduction of a five-dimensional theory. In STT one "forgets" about this fact and instead employs this coupling by hand. In JBD w is an arbitrary constant whereas in recent quintessence theories a dependence $w(\phi)$ has been proposed. A phenomenological potential $V(\phi)$ can be used to describe various cosmological scenarious [15]. Spherical reduction occurs similar to the case of Einstein gravity. Replacing the scalar curvature by (6), using the field redefinition (8) and setting D=4 we can integrate out the angular coordinates on S^2, i.e. θ, φ, to obtain the 2D action

$$\begin{aligned} S = \int_{M_2} d^2x\sqrt{-g}\bigg[\phi\left(XR + \frac{(\nabla X)^2}{2X} - \frac{1}{2}\right) + 2\nabla_\alpha\phi\nabla^\alpha X \\ -w(\phi)X\frac{(\nabla\phi)^2}{\phi} + XV(\phi)\bigg]. \end{aligned} \tag{11}$$

Here we have already performed a partial integration and divided by the overall factor 4π. It is now convenient to apply a conformal transformation

$$g_{\mu\nu} \to g_{\mu\nu}\phi^{-1} \ , \ \ X \to X\phi^{-1}, \tag{12}$$

together with a field-redefinition $\phi = A(Y)$, A being a solution of the ordinary differential equation

$$\frac{d\ln A}{dY} = \left(w(A) + \frac{3}{2}\right)^{-1/2} \tag{13}$$

that brings the action to the Einstein form

$$S_E = \int_{M_2} d^2x\sqrt{-g}\left[XR + \frac{(\nabla X)^2}{2X} - \frac{1}{2} - X(\nabla Y)^2 + \frac{X}{A(Y)}V(A(Y))\right]. \tag{14}$$

In this form the mixed term $\nabla_\alpha\phi\nabla^\alpha X$ disappears. However, in the case of interaction with matter a complicated nonminimal coupling to the STT field arises. In the matterless case the STT field is seen simply to play the role of an additional scalar field with proper (nonminimal) coupling to the geometric dilaton X in D=2.

2.2.3 Gowdy model

The four-dimensional (polarized) Gowdy metric [12]

$$ds^2 \;=\; e^{2a(t,\theta)}\left(dt^2 - d\theta^2\right) - X(t,\theta)\left(e^{Y(t,\theta)}d\sigma^2 + e^{-Y(t,\theta)}d\delta^2\right). \qquad (15)$$

describes a 4D spacetime that has 2 commuting Killing fields spanning a flat, spacelike isometry submanifold $T^2 \cong S^1 \otimes S^1$ (locally). Moreover it is assumed that the whole spacetime M_4 is compact. Performing the integration over the isometric coordinates σ, δ yields an effective two-dimensional action. For this reason we have to decompose the 4D scalar curvature $R^{(4)}$ into terms corresponding to T^2 and M_2, which is the complementary manifold, and terms produced by the embedding. This computation is done most conveniently in the vielbein frame[2] $ds^2 = \eta_{ab}e^a e^b - \left(e^2\right)^2 - \left(e^3\right)^2$. Quantities associated to T^2 or M_2 shall be assigned a tilde. We treat T^2 as two independent one-dimensional spaces S_1. Thus the relation between the vielbeine is given by

$$e^a = \tilde{e}^a,\;\; e^2 = \sqrt{X}e^{\frac{Y}{2}}\tilde{e}^2,\;\; e^3 = \sqrt{X}e^{-\frac{Y}{2}}\tilde{e}^3. \qquad (16)$$

Demanding vanishing torsion and metric compatibility on M_4, M_2 and T^2 the connection 1-form on M_4 is obtained:

$$\begin{aligned} \omega^a_{\ b} = \tilde{\omega}^a_{\ b} \quad &, \quad \omega^2_{\ 3} = \tilde{\omega}^2_{\ 3} = 0 \\ \omega^2_{\ a} = \left(\tilde{E}_a\sqrt{X}e^{\frac{Y}{2}}\right)\tilde{e}^2 \quad &, \quad \omega^3_{\ a} = \left(\tilde{E}_a\sqrt{X}e^{-\frac{Y}{2}}\right)\tilde{e}^3. \end{aligned} \qquad (17)$$

This is sufficient to calculate the scalar curvature

$$R^{(4)} = R - 2\frac{\Box X}{X} + \frac{1}{2}\frac{\nabla_\alpha X\nabla^\alpha X}{X^2} - \frac{1}{2}\nabla_\alpha Y\nabla^\alpha Y \qquad (18)$$

where R again is the scalar curvature of M_2. We can put this result into the 4D Einstein-Hilbert action and then integrate over the isometric coordinates while decomposing the measure as $\sqrt{-g_{M_4}} = X\sqrt{-g_{M_2}}$. The effective 2D action divided by the (finite) volume of T^2 reads

$$S = \int_{M_2} d^2x\sqrt{-g}\left[XR + \frac{(\nabla X)^2}{2X} - \frac{1}{2}X(\nabla Y)^2\right]. \qquad (19)$$

It is interesting to note that the dilaton X acquires the scaling factor of the 4D metric while the dilaton Y represents the gravitational degree of freedom

[2] Letters from the Latin alphabet denote vielbein indices, while Greek ones are reserved for coordinate indices.

of the Gowdy model. Clearly this action is of the Einstein form (3). It can be shown that its variation leads to the same EOM as the ones from the original 4D action when the symmetry is introduced there. This point is nontrivial as witnessed by the reduced action resulting from warped metrics in Einstein gravity [16].

From the Gowdy line-element alone it is not clear why to define as dilaton fields X and Y as in (15). In principle other gauge choices are possible. But in this representation the X-field alone carries the scale factor and hence the geometric information ("radius") of the Gowdy spacetime, while the Y-field represents a propagating degree of freedom ("graviton"). It is only in these variables (modulo trivial field redefinitions not mixing the dilatons) that the factorizability property, as defined below, is manifest.

3 Classification of TDT in 2D

In this section we will introduce useful notions, with respect to which we will classify TDT. As a word of warning we would like to emphasize the following point: Our definitions are not invariant under arbitrary field redefinitions.

However, for theories with only one dilaton coming from the higher-dimensional metric field redefinitions, leading to a mixing between the geometric dilaton and the "scalar field" dilaton in the new variables, are very inconvenient for two reasons: First, in general such transformations change the geodesics of testparticles[3] and hence the geometric properties of the manifold. Second, the quantization procedure used in [17,33] is spoilt by a mixing of geometric and matter variables[4].

For the case where both dilatons stem from the higher-dimensional metric such field redefinitions correspond to the choice of a different gauge for this metric and are therefore possible. Using this feature, by choosing a specific "gauge", we will show below that a general class of such models always fits nicely into our classification scheme.

3.1 Definitions

We start the classification with some new definitions which prove useful:

[3]The Christoffel symbols in the transformed geodesic equation $\ddot{x}^\alpha + \Gamma^\alpha{}_{\beta\gamma}(\tilde{X},\tilde{Y})\dot{x}^\beta\dot{x}^\gamma = 0$ become dependent on the new *matter* dilaton $\tilde{Y}$.

[4]In the following we will apply conformal transformations on such models. The conformal invariance (if any) then permits a "mixing" of the variables.

Definition 1: A TDT in the EF (3) is called *simple* iff $V_3^E = 0$.

Simple theories are models with no dynamical mixing between the dilaton fields and can be treated like a one-dilaton theory with dilaton field X and an additional scalar matter field Y coupled non-minimally in general. A main reason for highlighting such models is the possibility to bring them into a first order form (see below). This means that one can easily apply the quantization scheme developed in this framework [17]. Note that it is always possible to redefine the dilaton fields such that the diagonal term vanishes. E.g. one could use the nonlinear transformation $X \to \tilde{X}^\alpha \cdot f(\tilde{\phi})$, $\phi \to \tilde{X}^{1-\alpha} \cdot f^{-1}(\tilde{\phi})$; $\alpha = (w+1)/(w+3/2)$ to make spherically reduced JBD ((11) for $w = const.$) simple. However, as we have already mentioned, such a redefinition is only allowed if both dilatons stem from the higher-dimensional metric.

Definition 2: A TDT in any given form is called *factorizable* iff $V_1(X,Y) = f_1(X)g(Y)$ and $V_0(X,Y) = f_2(X)g(Y)$. We assume that (at least) X is part of a higher-dimensional metric, while Y can either be also part of this metric or a "true" scalar field.

Factorizable theories permit a simple geometrical interpretation of X as "classical dilaton field" in the 2D model, since there is a common Y-factor $g(Y)$ in front of the first two terms of (1). In the EF this property translates into $V_1^E = V_1^E(X)$.

In the following we discuss a class of models (including all models considered in this paper) where the original D-dimensional spacetime M_D contains one or two maximally symmetric, spacelike subspaces $S_{(1,2)}$(e.g. $S^1, S^2 \otimes S^3$) such that locally $M_D \cong M_2 \otimes S_{(1)} \otimes S_{(2)}$.

In the first case one dilaton arises from the metric and the other from the matter Lagrangian. These models are trivially factorizable (e.g. JBD, SRG).

In the second case both dilatons stem from the metric which can then be written in the form [18] [5]

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta - X^2[e^{2Y/d_1}d^2\Omega_{(1)} + e^{-2Y/d_2}d^2\Omega_{(2)}] \tag{20}$$

where $d^2\Omega_{(1,2)}$ are the metrics of the two subspaces with dimensions d_1, d_2 respectively, $g_{\alpha\beta}$ is the metric of the reduced two-dimensional spacetime and $X(x^\alpha), Y(x^\alpha)$ are the dilatons. The factors in the definition of Y are chosen

[5] As explained above this is just a specific choice of gauge for the metric of M_D.

such that the measure only depends on X which carries the conformal weight:

$$\sqrt{(-1)^{D+1}g_{M_D}} = \sqrt{-g_{M_2}}\sqrt{(-1)^{d_1+d_2}g_{(1)}g_{(2)}} \cdot X^{d_1+d_2} \,. \tag{21}$$

The reduced two-dimensional action divided by the volumes of the subspaces reads

$$\begin{aligned} S_2 \;&=\; \int_{M_2} \sqrt{-g_{M_2}} \cdot X^{d_1+d_2} \Bigg[R + (d_1+d_2)(d_1+d_2-1)\frac{(\nabla X)^2}{X^2} - \\ &\quad -(\frac{1}{d_1}+\frac{1}{d_2})(\nabla Y)^2 - \frac{R_{(1)}}{X^2}e^{-2Y/d_1} - \frac{R_{(2)}}{X^2}e^{2Y/d_2} \Bigg] \end{aligned} \tag{22}$$

where a $\Box X$ term has been partially integrated using the measure (21). $R_{(1,2)}$ are the (constant) scalar curvatures of the corresponding subspaces. As the kinetic term of the X-dilaton does not depend on Y we conclude that also this class of models is factorizable, containing polarized Gowdy ($T^2 \cong S^1 \otimes S^1$) and spherically reduced KKT ($S^1 \otimes S^2$). From (22) one can also see that these models are simple. Thus, by field redefinitions – corresponding to the choice of adapted coordinates in the higher-dimensional metric – it is always possible to bring them into the EF and make the mixed kinetic term vanish.

From JBD we know that it is conformally equivalent to Einstein gravity modulo the aforementioned problem of coupling to matter and a potential change of geodesic behavior. We will call such theories conformally related:

Definition 3: Two theories are called *conformally related*, iff there exists a conformal transformation (in the higher-dimensional theory) between them.

It is an interesting task to investigate whether a TDT given in the JF is conformally related to a simple model, since such models are particularly easy to treat and interpret. However, not all models allow a simplification through conformal transformations.

Definition 4: If a non-simple model is conformally related to a simple model we call it *conformally simple.*

By explicit calculation we will show below that all factorizable, non-simple models where the functions V_0, V_3 are monomials are conformally simple, provided that only one of the dilatons carries a conformal weight.

We would like to emphasize that conformally related theories represent dynamically inequivalent models in general. A simple example is the CGHS-model [19] which can either be introduced by complete spherical reduction

from an infinite-dimensional Einstein-Hilbert action (cf. eq. (9) for $D \to \infty$) or by the requirement of scale-invariance in the 2D action:

$$
\begin{aligned}
V_0^{CGHS} &= X, & V_1^{CGHS} &= 1/X, & V_2^{CGHS} &= 0, \\
V_3^{CGHS} &= 0, & V_4^{CGHS} &= X, & V_5^{CGHS} &= 0.
\end{aligned}
\tag{23}
$$

This action is invariant under a constant rescaling $X \to \lambda X$. Through an intrinsically 2D conformal transformation with a conformal factor $\Omega = X^{1/2}$ one can get rid of the V_1-term and the transformed theory describes flat spacetime. Thus the conformally related global structures are profoundly different: The Black Hole singularity of CGHS has disappeared. Also for any other theory important properties of the spacetime such as the 2D curvature and geodesic (in)completeness can be changed by a conformal transformation [20].

Despite of this, conformal transformations are frequently used in the literature on quantization of 2D dilaton gravity (cf. e.g. [21]) or JBD (cf. e.g. [22]) although by now even some of the proponents of this method [23] have (re)discovered this subtlety [24]. The issue of (in)equivalence of conformal frames has a long history of confusion, as pointed out in [13, 25] (see also references therein and references 28,29 of [26] for positive and negative examples).

It is necessary to bear in mind that most 2D models are dimensionally reduced theories which follow from a physically motivated higher-dimensional model. Another alternative is that they are merely toy models. In both cases a conformal transformation changing the global structure leads to a different theory. In the first case no longer a 2D equivalent of the original theory (the "correct" conformal frame is known) is described. In the second case, one could have started from the transformed toy model instead of introducing an "auxiliary" toy model (one could have introduced the "correct" conformal frame from the very beginning). Of course, from a technical point of view, conformal transformations are very useful and indeed will be employed amply below, if they only represent an intermediate step (especially in the context of a classical theory: For the quantum case the frame where the quantization is performed must be the "correct" one under all circumstances).

In the following we consider conformal transformations

$$
g_{\alpha\beta} \to g_{\alpha\beta}\Omega^{-2}, \quad \sqrt{-g} \to \sqrt{-g}\Omega^{-2}, \quad X \to X\Omega^{-\alpha}, \quad Y \to Y, \tag{24}
$$

assuming that X has a conformal weight of $\alpha \in \mathbb{R}^*$ and Y has conformal weight zero. This applies to models in the JF where only X stems from dimensional reduction. To include a larger class of models we generalize the

JF by allowing V_0 to be an arbitrary function of Y:

$$V_0(X,Y) = X v_0(Y)\ . \tag{25}$$

Since we want to answer the question whether a model is conformally related to a simple one we have to impose the condition $V_3^E = 0$ after the (higher-dimensional) conformal transformation. The choice of the conformal factor

$$\Omega = (v_0(Y))^{1/\alpha} \tag{26}$$

is necessary to bring the action in the EF. The first term in (1) as a consequence of the identity, valid under the conformal transformation (24),

$$R \to R\Omega^2 + 2\frac{\nabla_\gamma\nabla^\gamma\Omega}{\Omega} - 2\frac{\nabla_\gamma\Omega\nabla^\gamma\Omega}{\Omega^2} \tag{27}$$

produces the first term in the EF (2), plus further "kinetic" terms. Note that Ω must be C^1, manifestly positive and invertible with respect to Y. Its inverse will be denoted by

$$Y = f(\Omega). \tag{28}$$

3.2 Conformally simple TDT

In the following steps we will restrict ourselves to a smaller subset of TDT having the advantage of simplifying calculations drastically while still being general enough to include all "physical" models considered so far (and more). Using (27) and dropping a boundary term the action (1) after the conformal transformation (24) with conformal factor (26) becomes

$$\begin{aligned} S_c = &\int_{M_2} d^2x\sqrt{-g} \\ &\Bigg[XR - 2\nabla_\gamma\Omega\nabla^\gamma X + V_1\Omega^{-2\alpha}\left(\alpha^2X^2\frac{(\nabla\Omega)^2}{\Omega^2} - 2\alpha X\frac{\nabla_\gamma X\nabla^\gamma\Omega}{\Omega} + (\nabla X)^2\right) \\ &+V_2f'^2(\nabla\Omega)^2 - \alpha V_3Xf'\Omega^{-\alpha-1}(\nabla\Omega)^2 + V_3\Omega^{-\alpha}f'\nabla_\gamma X\nabla^\gamma\Omega + V_4\Omega^{-2}\Bigg]\ . \end{aligned} \tag{29}$$

Note that in $V_i(X,Y)$ one has to replace $X \to X\Omega^{-\alpha}$ and $Y \to f(\Omega)$ as defined by (28).

Conformal simplicity requires the vanishing of the mixed term $\nabla_\gamma Y\nabla^\gamma X$. This yields a first order differential equation for the function f,

$$f'(\Omega) = \frac{2}{V_3}\left(\alpha V_1(X\Omega^{-\alpha},\Omega)X\Omega^{-\alpha-1} + v_0(\Omega)\Omega^{-1}\right), \tag{30}$$

which already restricts the functions V_1, V_3 severely because the l.h.s. of (30) is X-independent by construction.

The convenient ansatz, to be used as of now,

$$V_1 = X^{-1} v_1(\Omega), \quad V_3 = v_3(\Omega) \tag{31}$$

is sufficient to satisfy (30) although not necessary. Next we impose factorizability on the original model which together with (31) implies

$$V_1(X,\Omega) = v_1 X^{-1}\Omega^{\alpha}, \quad v_1 \in \mathbb{R}. \tag{32}$$

Assuming monomiality for V_3 by

$$v_3(\Omega) = v_3\Omega^{\beta}, \quad \beta \in \mathbb{R}, \quad v_3 \in \mathbb{R}^* \tag{33}$$

the differential equation (30) establishes a four-parameter solution[6]

$$Y = f(\Omega) = 2\Omega^{\alpha-\beta}\frac{1+\alpha v_1}{v_3(\alpha-\beta)} =: c\Omega^{\gamma}, \quad \alpha \neq \beta. \tag{34}$$

This equation implies monomiality of $v_0(Y)$, too:

$$v_0(Y) = c^{-\delta}Y^{\delta}, \quad \delta := \frac{\alpha}{\alpha-\beta}. \tag{35}$$

Thus, with the assumptions made above $v_0(Y)$ is completely determined.

The resulting action may be written as

$$\begin{aligned} S_{cs} &= \int_{M_2} d^2x\sqrt{-g}\left[XR + v_1\frac{(\nabla X)^2}{X} + V_4\left(\frac{X}{v_0(Y)}, Y\right)v_0(Y)^{-2/\alpha}\right. \\ &\quad \left. + (\nabla Y)^2\left[V_2(\frac{X}{v_0(Y)}, Y) - \alpha\delta^2\frac{X}{Y^2}(2+v_1\alpha)\right]\right], \end{aligned} \tag{36}$$

which is conformally related to the original TDT action

$$\begin{aligned} S_{TDT} &= \int_{M_2} d^2x\sqrt{-g}\left[Xv_0(Y)R + v_0(Y)v_1\frac{(\nabla X)^2}{X} + V_4(X,Y)\right. \\ &\quad \left. + V_2(X,Y)(\nabla Y)^2 + v_3\frac{cv_0(Y)}{Y}\nabla_{\gamma}X\nabla^{\gamma}Y\right]. \end{aligned} \tag{37}$$

We recall the meaning of the four parameters: α is the conformal weight of the (geometric) dilaton X, δ (or β or γ) defines the power of the monomial

[6]The solution for the conformal factor of course only depends on two real parameters, c and γ, but the original action and the transformed one contain all four parameters α, β, v_1 and v_3.

$v_0(Y)$, v_1 and v_3 are constants entering the corresponding functions (c is defined in (34) and depends on all these constants).

Thus we have shown that a TDT satisfying (24)-(35) is conformally simple, provided that Y is positive everywhere. We emphasize that factorizability is conserved under this conformal transformation, as can be seen from (36), and hence is an independent property indeed.

The most important examples are STT, which are known to be conformally simple [2–4]:

$$\alpha = 2,\ \delta = 1 (\beta = 0),\ v_1 = 1/2,\ v_3 = 2\,. \tag{38}$$

The relation between the conformal factor and the STT field is $f(\Omega) = Y = \Omega^2$, a well-known result. Note that the whole class of STT is given by a single point in the four-dimensional parameter space of possible conformally simple actions with conformal factor given by (34). Thus, despite of the various restrictions which led to (34), the set of conformally simple theories described by the action (36), resp. (37), is very large.

It is straightforward to construct toy models which possess all possible combinations of factorizability and (conformal) simplicity.

4 First Order Formalism

Dilaton models that are (conformally) simple as well as factorizable have the important property that they may be written in an equivalent first order form [8]. This is manifest in the EF. In this case one of the dilatons (namely the scalar field Y) is disentangled from the gravitational sector, in the sense that no mixed kinetic term appears. The geometric part of the Lagrangian (including the dilaton X) can be brought to first order in its derivatives by introducing Cartan variables and auxiliary fields X^a. The zweibein basis is expressed in light-cone coordinates[7] $e^{\mp} = \frac{1}{\sqrt{2}}(e^0 \mp e^1)$. The invariant volume element in this frame is given by $d^2x\sqrt{-g} = d^2x(e) = e^- \wedge e^+$. The Levi-Civitá symbol $\varepsilon^{\bar{a}\bar{b}}$ is defined by[8] $\varepsilon^{-+} = -1$. The connection 1-form $\omega^{\bar{a}}{}_{\bar{b}}$ which is proportional to $\varepsilon^{\bar{a}}{}_{\bar{b}}$ becomes $\omega^{\bar{a}}{}_{\bar{b}} = -\omega \cdot \mathrm{diag}\,(1, -1)$. Thus the 2D scalar curvature can be written as $d^2x\sqrt{-g}R = -2d\omega$. According to the second reference of [8] we add the terms $X_{\bar{a}}T^{\bar{a}} = X^-(d+\omega)e^+ + X^+(d-\omega)e^-$ to the action where $T^{\bar{a}}$ is the torsion associated with the connection ω. The EF

[7] We choose a representation $(0,1) \to (-,+)$. Light-cone indices are adorned with bars.

[8] The ε-symbol in ordinary coordinates is defined by $\varepsilon^{01} = 1$.

action (2) divided by (-2) becomes equivalent to

$$S_{FO} = \int\limits_{M_2} \left[X^- (d+\omega) e^+ + X^+ (d-\omega) e^- + X d\omega + e^- \wedge e^+ V_1^E (X) X^- X^+ - \right.$$
$$\left. -\frac{1}{2} V_2^E (X,Y) \, dY \wedge *dY - \frac{1}{2} e^- \wedge e^+ \left(V_4^E (X,Y) + V_5^E (X,Y) f_m \right) \right], \quad (39)$$

where we have included also the matter term. The fields $X^\mp$ and X are determined from the EOM produced by the variation of the Cartan variables. The whole set of EOM derived from (39) is equivalent to the one obtained from the original action [27]. Actually $X^\pm$ and ω may be eliminated by *algebraic* EOM from (39).

For a theory in the EF the corresponding first order formulation has many advantages, especially at the quantum level, where e.g. the geometric degrees of freedom of SRG can be integrated exactly [17, 27]. Here we will only use one important result, namely the existence of a conservation law that can be derived in a particularly simple way in this context [9–11]. Taking appropriate linear combinations of the EOM derived from (39) with an integrating factor $I(X) = e^{-\int^X V_1^E (X') dX'}$ we obtain a relation of the type

$$I(X) \partial_\alpha \left(X^- X^+ \right) - I(X) \partial_\alpha X \left(V_1^E (X) X^- X^+ - \frac{V_4^E (X,Y)}{2} \right) + W_\alpha = 0. \quad (40)$$

Splitting the potential V_4^E into two terms $V_4^E = V_4^{(g)} (X) + V_4^{(Y)} (X,Y)$ we obtain the conservation law

$$\partial_\alpha \mathcal{C} = \partial_\alpha \mathcal{C}^{(g)} + W_\alpha = 0 \quad (41)$$

where $\mathcal{C}^{(g)} = X^- X^+ I(X) + \frac{1}{2} \int^X V_4^{(g)} (X') \, I(X') dX'$. From (41) the 1-form $W_\alpha = W_\alpha^{(Y)} + W_\alpha^{(m)}$ is trivially exact. Its separation into matter and Y-terms depends on the coupling function V_5^E that can have an arbitrary Y-dependence. The components $W_\alpha^{(Y)}$ are given by

$$W_\alpha^{(Y)} = I(X) \left[\frac{\partial_\alpha X}{2} V_4^{(Y)} (X,Y) + \right.$$
$$\left. + \frac{V_2^E (X,Y)}{(e)^2} \left\{ Y^- Y^+ (\partial_\alpha X) - (e) \left(Y^- X^+ + Y^+ X^- \right) \partial_\alpha Y \right\} \right] \quad (42)$$

where $Y^{\mp} = \varepsilon^{\alpha\beta} e^{\mp}_{\beta} (\partial_\alpha Y)$. The analogous expression for the matter part becomes

$$W^{(m)}_{\alpha} = I(X) \frac{V^E_5 (X,Y)}{2} \left[(\partial_\alpha X) f_m - (e) \varepsilon_{\alpha\beta} \left(X^- \frac{\partial f_m}{\partial e^-_\beta} + X^+ \frac{\partial f_m}{\partial e^+_\beta} \right) \right] . \tag{43}$$

It has been shown [11] that this conservation law is connected to the energy conservation of the model considered. More precisely, the geometric part $\mathcal{C}^{(g)}$ is proportional to a mass-aspect function $m_{eff}(r,t)$ which is the sum of the ADM mass and the energy fluxes given by the matter- and Y-contributions. Since we have not specified as yet the functions $V^E_1, V^E_2, V^E_4, V^E_5$ we have generalized that conservation law from EMKG to all factorizable (conformally) simple theories.

It is interesting to check its behavior under a (constant) Weyl-rescaling $g_{\alpha\beta} \to \lambda g_{\alpha\beta}$ in the higher-dimensional theory (taking into account the conformal weight of the geometric dilaton X). For general spherically reduced models we obtain a scaling weight

$$\mathrm{sw}(\mathcal{C})|_{SR} = D - 3, \tag{44}$$

where D is the higher dimension (e.g. D=4 for ordinary SRG), provided that the matter part and Y-part couple linearly to the dilaton X. In all other cases (the coupling to matter or Y is different or the potentials differ from SRG) the conserved quantity does not have a well defined conformal weight with respect to a global conformal transformation in the higher-dimensional theory (e.g. the CGHS model (23) contains minimally coupled scalar fields instead of linearly coupled ones).

5 Conclusions

In this paper we have investigated TDT produced by dimensional reduction. First we introduced two standard forms, namely the Einstein form (2) and the Jordan form (3), covering all models considered in this paper. The useful properties factorizability, simplicity and conformal simplicity have been defined. Since there seems to be still confusion in the literature (for a selected list of such papers cf. e.g. the review article [25]) we have emphasized the physical inequivalence of conformally related theories.

We could show that all investigated models can be derived by dimensional reduction of a spacetime with one or two maximally symmetric subspaces[9].

[9] In some cases like JBD one "forgets" about the origin of one of the dilatons. This manifests itself in setting its conformal weight equal to zero.

From this we conclude that:

- All these models are factorizable.
- Models with one geometric dilaton are at least conformally simple.
- Models with two geometric dilatons are simple (in adapted coordinates).

The second result was obtained by explicitly mapping a non-simple TDT in the JF (with V_0, V_3 monomials) onto a simple TDT in the EF through a conformal transformation. Thereby we could also show that factorizability is conserved under such a conformal transformation. It is not yet clear if theories with more complicated Killing-orbits are still factorizable. This could be the subject of further work.

Our subsequent investigations were restricted to the subclass of (conformally) simple factorizable theories. In the EF these models allow a first order formulation, and by straightforward application of previous work [9–11, 30] an absolute conservation law (41) could be established.

By investigating the scaling weight of the conserved quantity under global conformal transformations in the higher-dimensional theory for SRG the intuitively expected result (44) was obtained. Moreover it was clarified that a necessary condition for a definite scaling weight of the conserved quantity was linear coupling of the matter fields and the second dilaton Y to the (geometric) dilaton X, i.e. to the dilaton with a non-vanishing scaling weight.

Apart from the obvious applications (namely a 2D description of various higher-dimensional models considered in this paper) TDT serve as toy models for D=4 theories with two dilaton fields and as a basis for generalizations to models with more than two dilatons. Compactification of e.g. D=11 supergravity can yield two or more dilaton fields, and up to now no satisfactory cosmological theory with a single scalar field (which serves e.g. as inflaton *and* quintessence) is known. Here one may hope that – as in the case of the nonvanishing cosmological constant (or quintessence?) – further input may be provided by the enormously increasing amount of astrophysical data to be expected for the near future. If the need for more dilaton fields should arise we believe that similar structures in the classification of such models will appear.

Although the physically relevant TDT are related to dimensional reduction it could be of interest to focus on an intrinsically 2D treatment of TDT. Such an investigation would e.g. involve a conformal transformation where

Model	Form	V_1	V_2	V_3	V_4	V_5	Simple	DOF
1 dilaton	EF	$V_1(X)$	0	0	$V_4(X)$	0	Yes	0
Bos. String	EF	$V_1(X)$	$const.$	0	$V_4(X)$	1	Yes	N
SR EMKG	EF	$1/2X$	$-\kappa X$	0	$-1/2$	0	Yes	1
Pol. Gowdy	EF	$1/2X$	$-1/2X$	0	0	0	Yes	1
SR R^2	JF	$Y/2X$	0	2	$-Y/2 - XY^2/2$	0	Conf.	0
SR KKT	EF	$2/3X$	$-3X/2$	0	$-2\sqrt[3]{X}e^{-Y}$	$-X$	Yes	$1+m$
SR CCS	JF	$Y/2X$	$(3/2)X/Y$	2	$-Y/2$	0	Conf.	1
SR JBD	JF	$Y/2X$	$-wX/Y$	2	$-Y/2$	$-X$	Conf.	$1+m$
SR STT(J)	JF	$Y/2X$	$-wX/Y$	2	$-Y/2 + X\hat{V}(Y)$	$-X$	Conf.	$1+m$
SR STT(J)	EF	$1/2X$	$-(w+3/2)X/Y^2$	0	$-1/2 + X\hat{V}(Y)/Y^2$	$-X/Y^2$	Yes	$1+m$
SR STT(E)	EF	$1/2X$	$-(w+3/2)X/Y^2$	0	$-1/2 - X\hat{V}(Y)$	$-X$	Yes	$1+m$
SR STT(E)	JF	$Y/2X$	$-wX/Y$	2	$-Y/2 + X\hat{V}(Y)Y^2$	$-XY^2$	Conf.	$1+m$

Table 1: A representative sample of TDT and their properties. Abbreviations: SR stands for spherically reduced. DOF means (continuous physical) degrees of freedom (m denotes the number of matter DOF); N is the number of target space coordinates for the bosonic string. In the case of 1 dilaton models or the bosonic string by adjusting $V_1(X)$ and $V_4(X)$ one obtains a variety of models, among them the CGHS model [19], the Jackiw-Teitelboim model [28] and the Katanaev-Volovich model [29]. CCS means conformally coupled scalar in D=4. This model is the limit $w \to -3/2$ of JBD. In the entries of spherically reduced STT(X) minimal coupling to matter in the X-frame in D=4 has been assumed. All models are factorizable.

no conformal weight is attributed to both dilatons and suggest the introduction of a "true" Jordan frame, i.e. a conformal frame where no dilaton at all is coupled to the scalar curvature.

At the quantum level the next step should be a Hamiltonian analysis and BRST quantization. Similarities to the analysis of non-minimally coupled scalars interacting with a one-dilaton theory [31] which is based upon the simpler results obtained for the minimally coupled case [17, 32, 33] may well occur. In fact, for simple factorizable theories the constraint algebra is already known [31] and differs only slightly from the simpler algebra obtained in [32]. Non-simple, but conformally simple factorizable models fit into this theoretical frame only through a conformal transformation. Thus it will be an interesting task to investigate the action of a conformal transformation on the constraint algebra. This would provide a basis of (path integral) quantization of *all* conformally simple factorizable TDT.

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